**Example 1**

Find an interpolation formula for *ƒ*(*x*) = tan(*x*) given this set of known values:


\begin{align}
x_0 & = -1.5 & & & & & f(x_0) & = -14.1014 \\
x_1 & = -0.75 & & & & & f(x_1) & = -0.931596 \\
x_2 & = 0 & & & & & f(x_2) & = 0 \\
x_3 & = 0.75 & & & & & f(x_3) & = 0.931596 \\
x_4 & = 1.5 & & & & & f(x_4) & = 14.1014.
\end{align}


The Lagrange basis polynomials are:

\ell_0(x)={x - x_1 \over x_0 - x_1}\cdot{x - x_2 \over x_0 - x_2}\cdot{x - x_3 \over x_0 - x_3}\cdot{x - x_4 \over x_0 - x_4}
             ={1\over 243} x (2x-3)(4x-3)(4x+3)

\ell_1(x) = {x - x_0 \over x_1 - x_0}\cdot{x - x_2 \over x_1 - x_2}\cdot{x - x_3 \over x_1 - x_3}\cdot{x - x_4 \over x_1 - x_4}
             = {} -{8\over 243} x (2x-3)(2x+3)(4x-3)

\ell_2(x)={x - x_0 \over x_2 - x_0}\cdot{x - x_1 \over x_2 - x_1}\cdot{x - x_3 \over x_2 - x_3}\cdot{x - x_4 \over x_2 - x_4}
             ={3\over 243} (2x+3)(4x+3)(4x-3)(2x-3) 

\ell_3(x)={x - x_0 \over x_3 - x_0}\cdot{x - x_1 \over x_3 - x_1}\cdot{x - x_2 \over x_3 - x_2}\cdot{x - x_4 \over x_3 - x_4}
             =-{8\over 243} x (2x-3)(2x+3)(4x+3)

\ell_4(x)={x - x_0 \over x_4 - x_0}\cdot{x - x_1 \over x_4 - x_1}\cdot{x - x_2 \over x_4 - x_2}\cdot{x - x_3 \over x_4 - x_3}
             ={1\over 243} x (2x+3)(4x-3)(4x+3).

Thus the interpolating polynomial then is

 \begin{align}L(x) &= {1\over 243}\Big(f(x_0)x (2x-3)(4x-3)(4x+3) \\
& {} \qquad {} - 8f(x_1)x (2x-3)(2x+3)(4x-3) \\
& {} \qquad {} + 3f(x_2)(2x+3)(4x+3)(4x-3)(2x-3) \\
& {} \qquad {} - 8f(x_3)x (2x-3)(2x+3)(4x+3) \\
& {} \qquad {} + f(x_4)x (2x+3)(4x-3)(4x+3)\Big)\\
& = 4.834848x^3 - 1.477474x.
\end{align} 